CA[illegible]XAM - US[illegible] INF[illegible]RMATION

January 200[illegible]

MECHANICS

El[illegible] App Approach

1. Describe the kinematics (a) Co[illegible]
   (b) Constraints

2. Diagram with all of the [illegible] forces

3. Statics: $\sum F_x = 0$, $\sum F_y = 0$, $\sum \tau = 0$

4. Rig[illegible] Body-Free

   (a) Motion of C of M: $F = m\tilde{a}$

   (b) Rotation about C of M: $\tau = I\,\alpha$

5. [illegible] Body-Fixed Point 0
   Rotation about 0 $\tau_0 = I_0\,\alpha$

6. U[illegible]

7. Rot[illegible] System - [illegible] Euler's [illegible] Equation

   $\tau = \frac{d\,L}{dt}$ in the [illegible]

   becomes $\tau = \frac{d\,L}{dt} + \omega \times L$ in the rotating fra[illegible]

In the [illegible]

Lagran[illegible]

1. Ch[illegible] co-ordinates q_1, [illegible]

2. Express kinetic energy (T) in terms of the co-ordinates and [illegible] time derivatives $\dot{q}_1, \dot{q}_2$ [illegible]

3. Express the potential [illegible] in terms of the [illegible]

4. Write down the [illegible] L = T - V

5. Use the Lagrange equation

$$\frac{d}{dt}\left(\frac{d[illegible]}{d\dot{q}_i}\right) [illegible] \frac{d[illegible]}{dq_i} = 0$$

for each of the co-ordinates q_i as the equations [illegible]

ELECTRICITY AND MAGNETISM

1. Ga[illegible] $\oint$ [illegible] Q/ε_0 where Q is the [illegible] charge enclosed [illegible]
su[illegible] a dielectric me[illegible] $\oint \underline{D} \cdot d\underline{A} = Q$ [illegible] (D = [illegible])

2. Pote[illegible]ials:

Point charge: V = $Q/4\pi$[illegible]

Charged sphere, radius [illegible]

Surface potential: [illegible] $4\pi\varepsilon_0 a$

r > [illegible] $Q/4\pi\varepsilon_0 r$

Electric fi[illegible]: $\underline{E} = -\nabla V$ [illegible]

3. Magnetic Field:

Amperès Law $\oint \underline{B} \cdot d\underline{l} = \mu_0 I$

Long [illegible] $B = \frac{\mu_0 I}{2\pi [illegible]}$ (tangential)

Lo[illegible] [illegible] $\underline{v} \times \underline{B}$[illegible]

or $d\underline{F} = I\, d\underline{l}$ [illegible]

Lenz's L[illegible] $\frac{d[illegible]}{dt}$

Maxwell's Equations: [illegible] ρ, [illegible] div B = 0

curl B = μ_0 [illegible] $\frac{d[illegible]}{dt}$

THERMODYNAMICS

Ideal Gas:

$PV = nRT$ always

$PV^{\gamma} = $ const. for adiabatic change

$U = U(T)$ only

$dU = C_V dT$, $dH = C_p dT$

Central Equation:

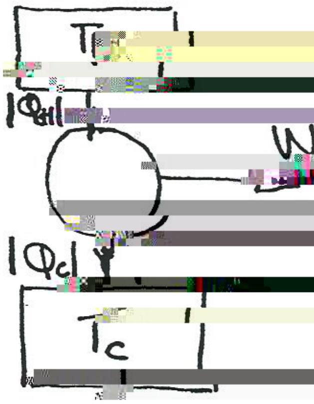
with 1st Law: $dU = dQ - dW$
with 2nd law: $dU = TdS - PdV$ (... $+ HdM + FdL + ...$)

Maxwell's Relations:

$(\partial T/\partial V)_S = -(\partial P/\partial S)_V$; $(\partial T/\partial P)_S = (\partial V/\partial S)_P$

$(\partial S/\partial V)_T = (\partial P/\partial T)_V$; $(\partial S/\partial P)_T = -(\partial V/\partial T)_P$

Carnot Engine:



$W = |Q_H| - |Q_C|$; $\eta = \frac{W}{Q_H}$

For Carnot (or all other reversible) engines:

$$\frac{|Q_H|}{|Q_C|} = \frac{T_H}{T_C}$$

i.e. $$\frac{Q_H}{T_H} + \frac{Q_C}{T_C} = 0$$

General reversible cycle:

$$\oint \frac{dQ}{T} = 0 \text{ or } \oint dS = 0$$

Note: For the reversible (Carnot) heat engine, the entropy is conserved.

SPECIAL THEORY OF RELATIVITY

Lorentz-Einstein Transformation

S is a frame at rest. S' has velocity v in the x direction

$x' = \gamma(x - vt)$ $\qquad$ $x = \gamma(x' + vt')$

$t' = \gamma(t - vx/c^2)$ $\qquad$ $t = \gamma(t' + vx'/c^2)$

$y' = y$ $\qquad$ $\gamma = 1/\sqrt{1 - v^2/c^2}$

$z' = z$

Lorentz contraction

L is the length of a rod at rest in S [illegible]

$$L = x_2 - x_1$$

L' is the length measured in S'; it is $x_2' - x_1'$ with x_1' and x_2' [illegible]

Use,

$$x_2 = \gamma(x_2' + vt')$$

$$x_1 = \gamma(x_1' + vt')$$

$$\therefore L' = L/\gamma$$

the moving [illegible] shorter.

Time Dilation

A clock in S is at rest [illegible] measure[illegible] constant. The equivalent interval in S' is

$$T' = t_2' - t_1' = \gamma T \text{ from above.}$$

That is T < T'. The time interval registered by the moving clock is less than the interval in the frame S'. The moving clock runs slowly.

Velocities

$$u_x' = x'/t' = (u_x - v)/(1 - u_x v/c^2)$$

$$u_y' = y'/t' = u_y/\gamma(1 - u_x v/c^2)$$

Doppler Shift

$$\omega' = \omega\sqrt{(1 - v/c)/(1 + v/c)}\ \text{receding}$$

$$\omega' = \omega\sqrt{(1 + v/c)/(1 - v/c)}\ \text{approaching}$$

Dynamics

Particles: $E^2 = E_0^2 + p^2c^2$

with [illegible] $E_0 = mc^2$, $p = \gamma mv$

Photon, Neutrino: $m = 0$, $E_0 = 0$, $E = pc$

Conserve momentum and energy.

STATISTICAL MECHANICS

Microstate: Defined by giv[illegible] in the system. Every accessible microstate is equally probable.

For [illegible] system comp[illegible] with number of accessible microstates [illegible] microstates Ω_2 for the system $\Omega = \Omega_1\Omega_2$. The temp[illegible] $\frac{1}{k_BT} = \frac{d\ln\Omega(E)}{dE}$

Boltzmann Factor: [illegible] probability that [illegible] system [illegible] microstate r [illegible] thermal equilibrium with a reservoir at temperature T is,

$$P_r = e^{-\beta E_r} / \sum_r e^{-\beta E_r}$$

Alternatively, [illegible]

$$dP(E) = g(E)dEe^{-\beta E} / \int g(E)dEe^{-\beta E}$$

where $\beta = 1/k_BT$, E_r is the energy of state r [illegible] $g(E)$ is the density of states.

Mean Value: For [illegible] tem in equilibrium at temperature T

$$\bar{a} = \sum_r a_r P_r$$

Partition Function: [illegible] $\sum_r e^{-\beta E_r}$

Then, $\bar{E} = -$ [illegible]

$P = kT($ [illegible]

$F = -kT \ln$ [illegible]

$C_V = -(d\beta/d$ [illegible]

Chemical Potential [illegible] $\mu =$ [illegible]

Classical and [illegible] **Quantum Gases**

Classical: All microstates are a [illegible] only one [illegible] (0011).

Fermi-Dirac [illegible]
(1011) [illegible] is good

Mean occupation [illegible] classical: f_s [illegible] $e^{-\beta\varepsilon_s}$

Bose: $f_s = \dfrac{1}{e^{\beta(\varepsilon_s - \mu)} - 1}$

Fermi: $f_s = \dfrac{1}{e^{\beta(\varepsilon_s - \mu)} + 1}$

QUANTUM MECHANICS

Schrödinger Equation

$$\hat{H}u = Eu \quad \text{(time-independent potential)}$$

$$u(r,t) = u(r)e^{-iEt/\hbar}$$

$$\psi = \sum_n c_n u_n; \quad \sum |c_n|^2 = 1$$

$$c_n = \int u_n^* \psi \, d\tau$$

New Basis

$$v_n = \sum c_m u_m$$

$$\int v_m^* v_n \, d\tau = \delta_{m,n}$$

Operators

$$\hat{x} = x, \quad \hat{p}_x = -i\hbar \frac{d}{dx}, \quad \hat{H} = V - \frac{\hbar^2}{2m}\frac{d^2}{dx^2}$$

Boundary Conditions

$$V = \text{finite:} \quad u_1 = u_2$$

$$V = \text{finite step:} \quad u_1 = u_2, \quad \frac{du_1}{dx} = \frac{du_2}{dx}$$

Time Dependence

e.g. $$\psi(x,t) = \frac{1}{\sqrt{2}}\left(u_\alpha e^{-iE_\alpha t/\hbar} + u_\beta e^{-iE_\beta t/\hbar}\right)$$

Hence interference

Perturbation Theory

$$E_m = E_m^0 + H'_{mm} + \sum_k \frac{\left|H'_{mk}\right|^2}{E_m^0 - E_k^0}$$

where $H'_{mk} = \int u_m^* H' u_k \, d\tau$

$$u_m = u_m^0 + \sum_k \frac{H'_{km} u_k^0}{E_m^0 - E_k^0}$$