2007 CAP EXA[illegible]

QUES[illegible]

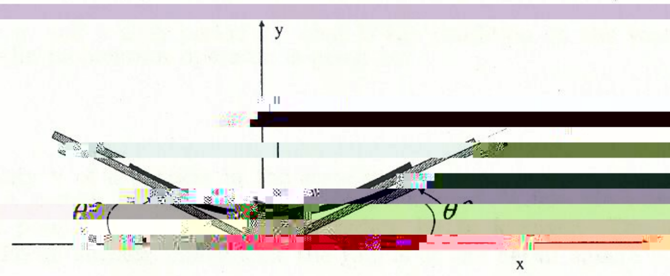
A rope [illegible] L [illegible] th [illegible] ine[illegible] (which [illegible]) [illegible] The rope has a uniform mass density, and its coefficient [illegible] friction with [illegible] platforms is [illegible]. The system is symmetric about [illegible]

(a) Dra[illegible] contact with the platforms [illegible]

(b) Given that the r[illegible] be down Newton's 1st law in component form.

(c) U[illegible] results from part [illegible] fraction of the rope f that does not touch the platforms.

(d) For wh[illegible] angle θ is the fr[illegible] f of r[illegible] touching the surface [illegible] max[illegible]



2 [illegible]

QUESTION 6

The [illegible] density in a [illegible] of depth H varies with depth as [illegible] where [illegible] ρ_0 is the density at the [illegible]rface, c is a constant [illegible] density va[illegible] amount [illegible] lighter fluid is [illegible] is on top [illegible] [illegible]ler [illegible] [illegible]ing for static stability of the [illegible]

(a) F[illegible] [illegible] static pressu[illegible] depth.

(b) Find the total mass of [illegible] of its centre of [illegible]

(c) [illegible] mass [illegible] or below the middepth (H[illegible] [illegible] uniform water? What is the [illegible] change in [illegible]tial e[illegible]

(d) Sugg[illegible] a density distribution that minimi[illegible] the po[illegible] the total [illegible] mu[illegible] cons[illegible] and the density must [illegible] value ρ_0 at the surface and a ma[illegible] the [illegible]

EZ [redacted]

2007 ... Exam

1. (a), (b) ... portion ... contact ... length L.

$f = \ldots$

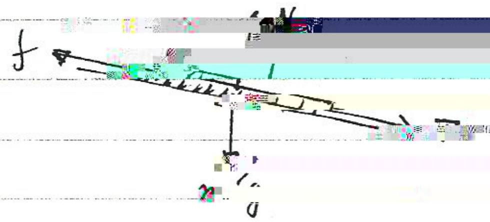
A... only ... tension ...

FBD for ...

$2T \sin \ldots$

$\ldots \lambda = $ mass/unit length

For the portion in contact with ...



$f = T + m g \sin\theta = T + \frac{1}{2}\lambda L g (\ldots)$

(c) If ... not ... we require $f \leq \mu N$

Considering ... which ... the maximum possible weight of the ...

can be ... $f_{max} = \mu N = N$ ...

$\therefore \ldots T \ldots$

$\ldots$

$f(\ldots) = \ldots$

$$f = \frac{\cos^2\theta \sin^2\theta - \sin^2\theta}{\cos^2\theta \sin^2\theta - \sin^2\theta}$$

$$= \frac{\sin 2\theta + \cos 2\theta - 1}{\sin 2\theta + \cos 2\theta + 1} \equiv \frac{g-1}{g+1}$$

[illegible]

To maximize f

$$f' = \frac{g'}{g+1} - \frac{(g-1)g'}{(g+1)^2} = \frac{2g'}{(g+1)^2} = 0$$

$$g' = 2\cos 2\theta - 2\sin 2\theta = 0$$

$$\tan 2\theta = 1$$

$$\therefore 2\theta = \frac{\pi}{4}$$

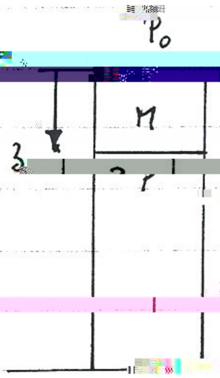
$$\theta = \frac{\pi}{8} \quad (22.5°)$$

$$f_{MAX} = \frac{\sqrt{2}-1}{\sqrt{2}+1} = 0.17$$

D[illegible] function of the cone which can be [illegible] 57° [illegible] results [illegible] the special case $\mu = 1$

200? CAP Exam

(a)



The pressure [illegible] at depth z:

$$PA = P_0 A + Mg$$

where $M = \int_0^z \rho \, [illegible]$

$$= \rho_0 A\left(z + \tfrac{1}{2} c z^2\right)$$

$$\therefore \; P(z) = P_0 + \rho_0 g z\left(1 + \tfrac{1}{2} c z\right)$$

(b) $M(H) = \rho_0 A H\left(1 + \tfrac{1}{2} c H\right)$

$$\frac{M}{A} = \rho_0 H\left(1 + \tfrac{1}{2} c H\right)$$

$$\bar{z} = \frac{\int_0^H z \, dm}{\int_0^H dm} = \frac{\int_0^H z \, \rho_0 (1 + cz) \, dz}{\int_0^H \rho_0 (1 + cz) \, dz}$$

$$= \frac{\tfrac{1}{2} H^2 + \tfrac{1}{3} c H^3}{H + \tfrac{1}{2} c H^2}$$

$$= \tfrac{1}{2} H \, \frac{1 + \tfrac{2}{3} c H}{1 + \tfrac{1}{2} c H}$$

$> \tfrac{1}{2} H$ [illegible]

$\therefore$ the centre of mass [illegible]

$\bar{\rho} = \frac{M}{V} = \frac{\rho_0 A H (1 + \frac{1}{2} c H)}{[illegible]} = \rho_0 (1 + \frac{1}{2} c H)$ [illegible]

Take $z = H$ as the zero of potential energy. The potential energy of an element at [illegible] is

$$dU = [illegible]\, g\, (H - z)$$

$$= \rho(z)\, A\, dz\, g\, (H - z)$$

For the original liquid,

$$U = \int_0^H \rho_0 (1 + cz)\, A g\, (H - z)\, dz$$

$$= \rho_0 A g \left(H^2 + \tfrac{1}{2} c H^3 - \tfrac{1}{2} H^2 - \tfrac{1}{3} c H^3 \right)$$

$$= \tfrac{1}{2} \rho_0 A g H^2 \left(1 + \tfrac{1}{3} c H \right)$$

For the mixed liquid,

$$\bar{U} = \int_0^H \bar{\rho} A g\, (H - z)\, dz$$

$$= \bar{\rho} A g \left(H^2 - \tfrac{1}{2} H^2 \right)$$

$$= \tfrac{1}{2} \bar{\rho} A g H^2$$

$$= \tfrac{1}{2} \rho_0 A g H^2 \left(1 + \tfrac{1}{2} c H \right) [illegible]$$

[illegible] $\bar{U}$ [illegible]

This is consistent [illegible]

(d) [illegible]

Then we should [illegible]

further. This [illegible]

density increase [illegible]

$$\rho(z) = \rho_0 (1 + c' z^n) \quad [illegible]$$

The [illegible]

$$M = \int_0 \rho_0 (1 + c z^n) \ldots [illegible]$$

$$= \rho_0 A ([illegible] \ c' \ [illegible]^{n+1})$$

$$= \rho_0 A H (1 + \frac{c'}{[illegible]} H^{n})$$

If this is to be held fixed we [illegible]

$$1 + \frac{c'}{[illegible]} H^{n} = 1 + \frac{c}{2} [illegible]$$

$$\text{i.e.} \quad c' = \frac{n+1}{2} c$$

$$\therefore \rho(z) = \rho_0 (1 + \frac{n+1}{2} c z^n)$$

One could then [illegible]

~~[illegible]~~ minimize [illegible]